$$\therefore \frac{1+at+bu}{\sqrt{[1+a^2+b^2]}}=\frac{1+ct+du}{\sqrt{[1+c^2+d^2]}}=\frac{1+ft+gu}{\sqrt{[1+f^2+g^2]}} \text{ determines } t \text{ and } u.$$

$$e^2=\tan^2\theta=\frac{(t-a)^2+(u-b)^2+(au-bt)^2}{(1+at+bu)^2},\ e_1^2=\frac{(t-a)^2+(u+b)^2+(au+bt)^2}{(1+at-bu)^2},$$

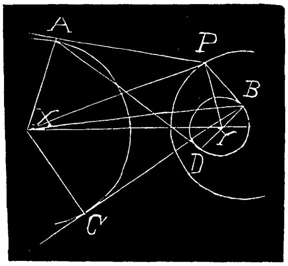
$$e_2^2=\frac{(t+a)^2+(u-b)^2+(au+bt)^2}{(1-at+bu)^2},\ e_3^2=\frac{(t+a)^2+(u+b)^2+(au-bt)^2}{(1-at-bu)^2}.$$

145. Proposed by FRANK GIFFIN, Graduate Student, State University, Boulder, Col.

If A and B be the points of contact, upon two circles X and Y, of tangents drawn from any point of their circle of similitude, then the tangent from A to Y is equal to the tangent from B to X. [From *Casey's Sequel to Euclid*, Part I., page 144.]

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let P be any point on the circle of similitude, AP, BP the tangents from P to X and Y, respectively. AD, BC the tangents from A and B to Y and X, respectively.



Let $AX=R$, $BY=r$. Let $AD=a$, $BC=b$, $AP=c$, $BP=d$, $PX=m$, $PY=n$.

$\angle APX=\angle BPY$ since P is on circle of similitude. $\therefore \angle APY=\angle BPX=\theta$.

Also, $c:d=R:r$. $\therefore d=cr/R \ldots.(1)$.

$m:n=R:r$. $\therefore n=mr/R\ldots.(2)$.

$a^2=AY^2-r^2=c^2+n^2-2cn\cos\theta-r^2$

$=c^2+m^2r^2/R^2-(2cmr/R)\cos\theta-r^2\ldots.(3)$.

$b^2=RX^2-R^2=d^2+m^2-2dm\cos\theta-R^2=c^2r^2/R^2+m^2-(2cmr/R)\cos\theta-R^2\,..(4)$.

$(3)-(4)$ gives $R^2(a^2-b^2)=(c^2-m^2+R^2)(R^2-r^2)$.

But $c^2+R^2=m^2$.

$\therefore a^2-b^2=0$. $\therefore a=b$.

CALCULUS.

106. Proposed by M. C. STEVENS, M. A., Professor of Higher Mathematics, Purdue University, Lafayette, Ind.

$$\int_0^\pi \frac{\cos rx dx}{1-2a\cos x+a^2}=\frac{\pi a^r}{1-a^2}.$$

[*Williamson's Integral Calculus*, 6th Edition, page 174.]

Solution by WILLIAM HOOVER, A.M., Ph.D., Professor of Mathematics and Astronomy, Ohio University, Athens, Ohio.

In Todhunter's *Plane Trigonometry*, 3d Edition, Art. 309, we have

$$\cos\alpha+a\cos(\alpha+\beta)+a^2\cos(\alpha+2\beta)+\ldots\ a^{n-1}\cos[\alpha+(n-1)\beta]$$